

Time allowed: 3 Hours

Max. Marks: 30

NOTE: Attempt five questions in all, selecting atleast two questions each Unit. All questions carry equal marks.

x-x-x

UNIT - I

1. (a) Let $f: \mathbf{R}^2 \rightarrow \mathbf{R}$ be defined by $f(x, y) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational.} \end{cases}$

Show that $\lim_{(x,y) \rightarrow (a,b)} f(x, y)$ for any point (a, b) does not exist.

- (b) Examine the function $f(x, y) = \frac{xy^3}{x^2 + y^2}$, $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$ for continuity at $(0, 0)$.

2. (a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$.

- (b) If V is a function of two variables x and y and $x = r \cos \theta$, $y = r \sin \theta$, prove that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 V}{\partial \theta^2} + \frac{1}{r} \frac{\partial V}{\partial r}$$

3. (a) Show that $f(x, y) = \sin x + \cos y$ is differentiable at every point of \mathbf{R}^2 .

- (b) Show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$, where

$$f(x, y) = \begin{cases} x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right) & \text{if } xy \neq 0 \\ 0 & \text{if } xy = 0. \end{cases}$$

4. (a) Find the directional derivative of $\phi = x^2 y^2 z^2$ at the point $P(1, 1, -1)$ in the direction of tangent to the curve $x = e^t$, $y = 2 \sin t + 1$, $z = t - \cos t$ at $t = 0$.

- (b) Show that $\nabla \cdot (\vec{f} \times \vec{g}) = \vec{g} \cdot (\nabla \times \vec{f}) - \vec{f} \cdot (\nabla \times \vec{g})$.

UNIT - II

5. (a) State and prove Euler's theorem on homogeneous functions of two variables.

- (b) Expand $\tan^{-1} \frac{y}{x}$ in the neighbourhood of the point $(1, 1)$.

6. (a) If u, v, w are the roots of the equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$, prove that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = -\frac{2(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$$

- (b) Show that the functions $u = x + y - z$, $v = x - y + z$, $w = x^2 + y^2 + z^2 - 2yz$ are not independent of one another. Also find the relation between u, v and w .

P.T.O.

(2)

7. (a) Find the extreme values of the function $f(x, y) = 12x^2 + 8xy + y^2 + 8x^3$.
- (b) Find the shortest distance from the origin to the hyperbola $x^2 + 8xy + 7y^2 = 225, z = 0$.
8. (a) Find the envelope of the family of the curves $\frac{a^2}{x} \cos \theta - \frac{b^2}{y} \sin \theta = c$ for different values of θ .
- (b) If ρ_1 and ρ_2 are the radii of curvature at the corresponding points of a cycloid and its evolute, prove that $\rho_1^2 + \rho_2^2 = \text{constant}$.

x-x-x

Dr. Himanshu Singla

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